Study of graph based optimization techniques intermodal connectivity in Kolhapur's public transportation

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ABSTRACT

In this paper we have to discussing the Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP). Here we comparing the number of algorithms such as Vector Path Routing (VPR), Dijkstra's Algorithm and A* Search. And also mention the case study that is a route from the Western India Institute of Neurosciences to Aster Aadhar Hospital. By analysing multiple optimization strategies, this research aims to improve mobility, reduce travel costs, and minimize delays.

KEYWORDS : Graph, path, Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP), Vector Path Routing (VPR), Dijkstra's Algorithm, A* Search, and Prim's Algorithm.

INTRODUCTION

The ability to travel efficiently is a crucial aspect of human activity, significantly influencing our daily lives. However, this need for mobility often leads to traffic congestion, which is a common issue that disrupts smooth travel experiences. Traffic congestion has far-reaching effects, including delays in road transportation, increased pollution levels, higher travel costs, and interruptions on routes that may seem minor. Many travellers find themselves stuck midway through their journeys, wasting valuable time and resources due to a lack of reliable and comprehensive traffic information. Anticipatory awareness of traffic conditions is essential, as it empowers individuals to select the most efficient routes available.

In logistics, transportation, and network optimization, routing challenges such as the Vehicle Routing Problem (VRP) and the Traveling Salesman Problem (TSP) present significant obstacles that need to be addressed. Effectively solving these routing problems is vital for enhancing overall productivity, reducing operational costs, and optimizing resource allocation across numerous real-world applications.

In this work we try to develop the various methods and algorithm for addressing routing transporting challenges, including A*, Dijkstra, and Bellman-Ford. These algorithms form the backbone of conventional hard computing techniques, providing predictable solutions based on strict mathematical principles. Also we study complexities of routes and routing problems within the context of road transportation systems, investigating a wide range of route optimization techniques that incorporate both soft and hard computing strategies.

PRELIMINARIES

- **Graph[8]:** A graph is formed by vertices and edges connecting the vertices. Formally, the graph is the pair of sets (V,E) is a set of edges form by Pairs of vertices.
- **Simple graph[8]:** A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.

- **Weighted Graph[8]:** Each edge has an associated weight or cost w(u ,v), which represents the cost to move from vertex u to vertex v.
- **Path[8]:** A path is a sequence of vertices connected by edges. The path cost is the sum of the weights of its edges.
- **Shortest Path[8]:** The shortest path from a source vertex s to a target vertex v is the path with the minimum total weight.
- **Path weight[8]:** The **path weight** is the sum of the edge weights in the path.
- **Cycle[8]:** A **cycle** in a graph is a path that starts and ends at the same vertex with **no repeated edges or vertices.**

MAIN RESULT

1) VRP (Vehicle Routing Problem)[9]

The concept of VRP was first introduced in 1956 by researchers Ramser and Dantzig, who developed strategies to optimize gasoline supply routes. This marked the beginning of mathematical approaches to solving routing issues. The main aim of the VRP is to reduce the total transportation costs. This includes minimizing the distance travelled by the vehicles and ensuring that each client is served efficiently, ideally only once.

Algorithm for VPR (Vector Path Routing)[9]

Step 1: Identify key- places (nodes) along the route.

Step 2: Draw a graph – For representing nodes and the distances between them.

Step 3: Apply VPR formula-

Cost (u ,v) = d(u, v) + h(v)

Where, d(u ,v) = distance between nodes u and v.

h(v)= heuristic estimate (we can assume it to be straight-line distance to end node)

assume, h(v)= 0.

Step 4: Finally got Shortest Path between different nodes.

2) TSP (Travelling Salesman Problem)[7]

The Traveling Salesman Problem (TSP) revolves around finding the optimal path for a salesperson, starting from a specified origin, visiting multiple locations in sequence, and returning to the starting point. Key points include:

Algorithm for TSP (Travelling Salesman Problem)[7]

Step 1: Select 6 Key Locations (Nodes)

Step 2: Draw the distance Table

Step 3: apply the TSP Formula

$$\text{TSP}_{\text{total}} = \min_{\text{permutations}} \sum_{i=1}^{n} d(p_i, p_{i+1}) + d(p_n, p_1)$$

Step 4: Got the final result (Min Distance).

3) Dijkstra's algorithm[10]

Dijkstra's algorithm is a way to find the shortest path between two points in a network, like a map with cities or a metro system with stations. It works step by step to figure out the quickest route from the starting point to the destination.

Algorithm for Dijkstra's algorithm

Step 1: Define Nodes (Places on Map)

Step 2: Draw the distance Table (Weighted Graph in km, approximated)

Step 3: Apply the Dijkstra's Algorithm Formula

Initialize distances:

dist[source]=0, dist[others]=∞

Update neighbours:

dist[v]=min(dist[v],dist[u]+weight(u,v))

Step 4: Got the final result (shortest distance)

4) ALT (A* search Landmarks and Triangle inequality)[10]

The ALT algorithm is a pre-processing method aimed at optimizing Dijkstra's algorithm for computing shortest paths in extensive road networks by selecting specific nodes known as landmarks for faster computations. A* is a search algorithm that finds the least-cost path from a start node to a goal node, using a cost function that combines:

Algorithm for A* algorithm

Step 1: Define Nodes (Places)

Step 2: Distance Table (g(n), estimated in km)

Step 3: Heuristic Table (h(n), estimated straight-line distance to F)

Step 4: A* Search (f(n) = g(n) + h(n))

g(n): Cost from start node to current node

h(n): Heuristic (estimated cost from current node to goal)

f(n): Total estimated cost of the path through node n

Step 5: Got the final result.

***Discussion and Comparison between different algorithms:**

Here we find the best (shortest) path from western India Institute of neurosciences to Aster Aadhar Hospital in Kolhapur by using above the all algorithms.

Node	Location
A	Western India Institute (Start)
B	Mahavir College Chowk
C	Shahupuri
D	Sayaji Hotel
E	Tarabai Park
F	Aster Aadhar Hospital (End)

Method	Path	Shortest distance
VPR (Vector Path Routing)	A-B-D-E-F	4.7
TSP(Travelling Salesman Problem)	A-B-C-D-E-F-A	9.4
Dijkstra’s Algorithm	A-B-C-D-E-F	5.4
A* Algorithm	A-B-C-D-E-F	5.4

CONCLUSION

We find the best (shortest) path from western India Institute of neurosciences to Aster Aadhar Hospital in Kolhapur by using VPR (Vector Path Routing). Also we observe that VPR (Vector Path Routing) method gives the shortest path and TSP(Travelling Salesman Problem) method gives largest path but its starting and end point are same. Both Dijkstra’s algorithm and the A* algorithm are widely used for finding the shortest paths in graphs, but they differ in efficiency, applicability, and overall effectiveness.

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